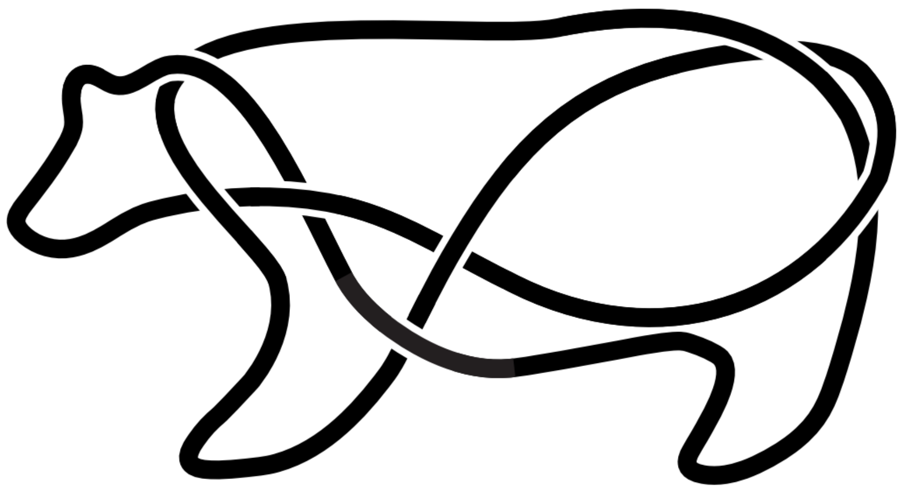
Berkeley Math Tournament 2025

Algebra Test



November 8, 2025

Time limit: 60 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No calculators. Protractors, rulers, and compasses are permitted.

- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2}+\frac{1}{3}$, but you do not need to evaluate large powers such as 7^8.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as π in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.
- For fractions, both improper fractions and mixed numbers are acceptable.

1. Real numbers x, y, and z satisfy
$$5x + 6y + 7z = 0,$$
$$6x + 7y + 8z = 1.$$
Compute $65x + 66y + 67z$.

2. A hat initially contains five slips labeled with the numbers 1, 2, 3, 4, and 5 (one slip per number). Every minute, Jessica adds a new slip labeled with the number 20 to the hat. How many slips are in the hat when the average of the numbers in the hat is 5 times the initial average?

3. An arithmetic sequence is a sequence of numbers where the difference between any two consecutive terms is the same. Harsh writes an arithmetic sequence where the ratio of the second term to the first term is 4, and the sum of the third and fourth terms is 34. Compute the second term of Harsh's sequence.

4. Let $f(x) = (2x^2 + 7x + 3)(x^2 - 2x - 15)$. The graph of $f(x)$ in the coordinate plane is translated 4 units to the right and then rotated about the origin by 180 degrees. The resulting graph is the graph of the function $g(x)$. Compute the sum of the distinct roots of $g(x)$.

5. Complex numbers a, b, and c satisfy the equations
$$a^3 + b^3 = 2 - 3ab(a + b),$$
$$b^3 + c^3 = 16 - 3bc(b + c),$$
$$c^3 + a^3 = 54 - 3ca(c + a).$$
Compute the sum of all distinct possible real values of $a + b + c$.

6. Compute the sum of all distinct real numbers x satisfying $|x| \leq \frac{1}{2}$ and
$$\sin\left(\log_{10}\left(\frac{1}{x} + \frac{1}{x^2}\right)\right) = 0.$$
Note that the argument of the sine function is assumed to be in radians.

7. Let $\omega = e^{\frac{2i\pi}{5}}$. Compute
$$\prod_{i=1}^{5} \prod_{j=i+1}^{5} (\omega^i - \omega^j)^2.$$

8. Let $a = \log_6(30)$ and $b = \log_{15}(24)$. Then
$$\frac{2ab + 2a - 1}{ab + b + 1} = \log_m(n)$$
for positive integers m and n such that m is as small as possible. Compute $m + n$.

9. Distinct complex numbers a, b, and c each have integer real part and satisfy the equations
$$ab + bc + ca = 300,$$
$$(a - b)^5 + (b - c)^5 + (c - a)^5 = 0.$$
Compute the sum of the three least possible positive real values of abc.

10. Complex numbers $a_1, a_2, \ldots, a_{10}$ satisfy the following property for each $1 \leq k \leq 10$:

$$\sum_{i=1}^{10} a_i^9 = 10a_k^{10} + a_k^9 - 2^9.$$

Given $a_1a_2 \cdots a_{10} = -\frac{m}{n}$ for relatively prime positive integers m and n, compute the sum of the digits of $m + n$.