Berkeley Math Tournament 2025

Discrete Tiebreaker

November 8, 2025

Time limit: 15 minutes.

Instructions: This tiebreaker contains 3 short answer questions. All answers must be expressed in simplest form unless specified otherwise. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.

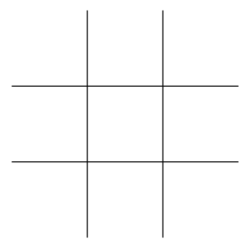
No calculators. Protractors, rulers, and compasses are permitted.

- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2} + \frac{1}{3}$, but you do not need to evaluate large powers such as 7^8.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as π in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.
- For fractions, both improper fractions and mixed numbers are acceptable.

Time limit: 15 minutes.
Instructions: This tiebreaker contains 3 short answer questions. All answers must be expressed in simplest form unless specified otherwise. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.
No calculators. Protractors, rulers, and compasses are permitted.

1. How many ways are there to place 3 identical $\bigcirc$ on a 3×3 tic-tac-toe board according to the following rules?

   - Each $\bigcirc$ is in a different cell.
   - Not all three $\bigcirc$ are in the same row, column, or diagonal.



2. Find the sum of all three-digit positive integers N such that N plus the reverse of N is a perfect cube. For example, the reverses of 245 and 370 are 542 and $073 = 73$, respectively.

3. A subset T of $S = \{1, 2, \ldots, 2025\}$ is *generational* if for every integer $1 \leq j \leq 2025$, there exist integers $c_1, c_2, \ldots, c_{2025}$ such that

$$\sum_{n \in T} nc_n \equiv j \pmod{2025}.$$

   What is the size of the largest subset T that is **not** generational?